MARK: $\dfrac{}{98}$ ..... %
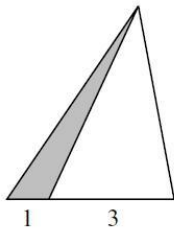
MATHS 2014

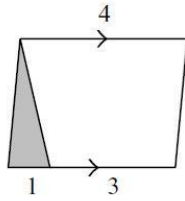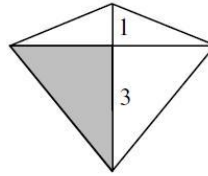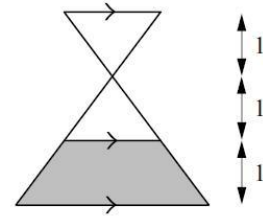# *HARD*

SELECTED

QUESTIONS

(Installment #1)

Q1
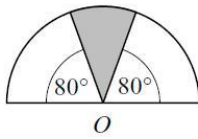

Diagram 1 Diagram 2 Diagram 3 Diagram 4

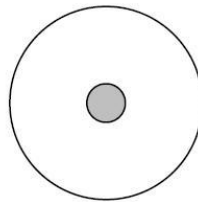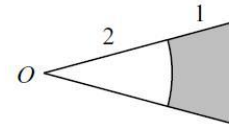Diagram 5 Diagram 6 Diagram 7

You have to work out the percentage of each shape that's shaded. For diagram 6, the radii of the two circles are 1 and 5. [13]

Q2

15

(b)
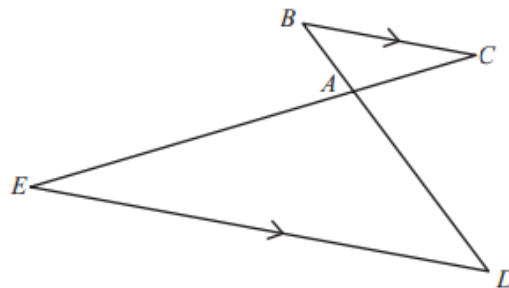
BAD and CAE are straight lines and BC is parallel to ED.

$\overrightarrow{BA} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $\overrightarrow{ED} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$ and $\overrightarrow{BA} = \frac{1}{4}\overrightarrow{BD}$.

(i) Describe fully the **single** transformation that maps triangle ABC onto triangle ADE.

Answer ....

....[2]

Q3

19 The diagram shows the metal cover for a circular drain.
Water drains out through the shaded sections.
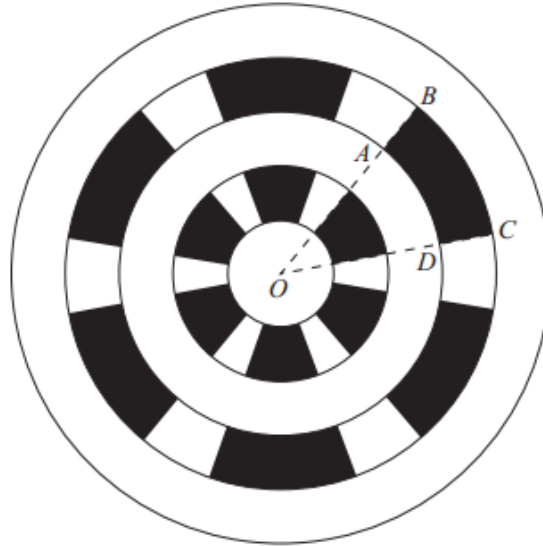
O is the centre of circles with radii 1 cm, 2 cm, 3 cm, 4 cm and 5 cm.
The cover has rotational symmetry of order 6 and $B\hat{O}C = 40°$.

(a) Calculate the area of the shaded section $ABCD$, giving your answer in terms of π.

Answer ....cm^2 [2]

(b) The total area of the metal (unshaded) sections of the cover is $\frac{55}{3}\pi\,\text{cm}^2$.

    (i) Calculate the total area of the shaded sections, giving your answer in terms of π.

           Answer ....cm^2 [1]

    (ii) Calculate the fraction of the total area of the cover that is metal (unshaded).
           Give your answer in its simplest form.

           Answer ....[1]

Q4

21 R is directly proportional to the **cube** of p.
When $p = 2$, $R = 24$.

    (a) Find the formula for R in terms of p.

           Answer $R =$ ....[1]

(b) Find the value of p when $R = 192$.

Answer $p =$...................................... [2]

(c) Which of the diagrams below represents the graph of R against p?

| Diagram 1 | Diagram 2 | Diagram 3 | Diagram 4 | Diagram 5 |
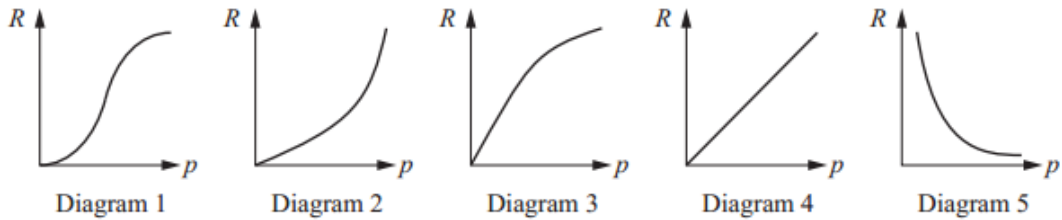
Answer Diagram .... [1]

Q5   **15**   This figure has rotational symmetry of order 3.
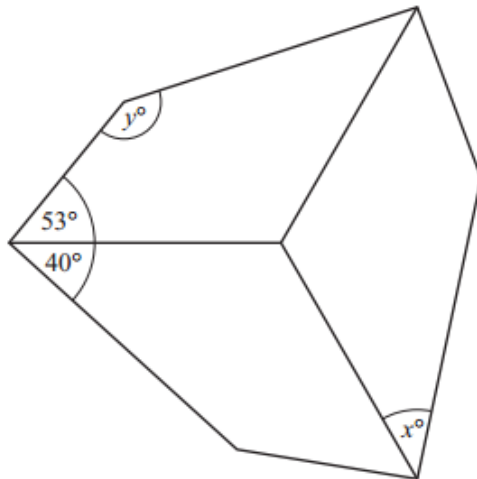
(a) How many lines of symmetry does the figure have?

Answer .... [1]

(b) Find x.

Answer $x =$ ....[1]

(c) Find y.

Answer $y =$ ....[1]

Q6

19 [Volume of a cone $= \frac{1}{3}\pi r^2 h$]

Cone 1 has radius $2x$ cm and height $7x$ cm.

Cone 2 has radius x cm and height $4x$ cm.

Find an expression, in terms of π and x, for the **difference** in the volume of the two cones.
Give your answer in its simplest form.

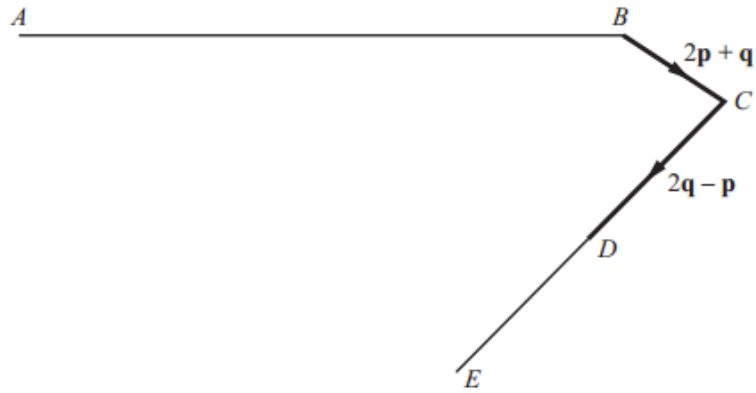*Answer* ....cm^3 [3]

21

In the diagram, $\overrightarrow{BC} = 2\mathbf{p} + \mathbf{q}$, $\overrightarrow{CD} = 2\mathbf{q} - \mathbf{p}$ and D is the midpoint of CE.

(a) Express, in its simplest form, in terms of $\mathbf{p}$ and/or $\mathbf{q}$

(i) $\overrightarrow{CE}$,

Answer ....[1]

(ii) $\overrightarrow{BE}$.

Answer ....[1]

(b) Given that $\overrightarrow{AB} = k\mathbf{p}$, express $\overrightarrow{AE}$ in terms of k, $\mathbf{p}$ and $\mathbf{q}$.

Answer ....[1]

(c) Given that AE is parallel to BC, find k.

Answer $k = $ ....[1]

8   (a)
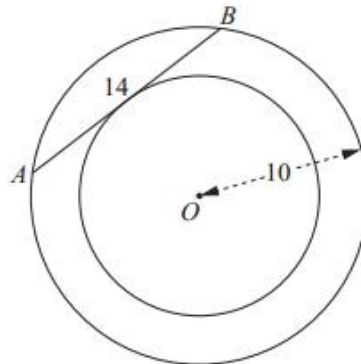
In the diagram, the circles each have centre O.
AB is a chord of the larger circle and also a tangent to the smaller circle.
$AB = 14$ cm and the radius of the larger circle is 10 cm.

Find the radius of the smaller circle.

Answer   .... cm [3]

(b)
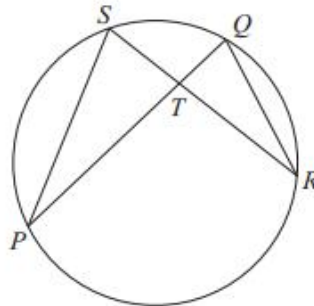
In the diagram, PQ and RS are chords of a circle that intersect at T.

(i)   Show that triangles PST and RQT are similar.

[3]

(ii)
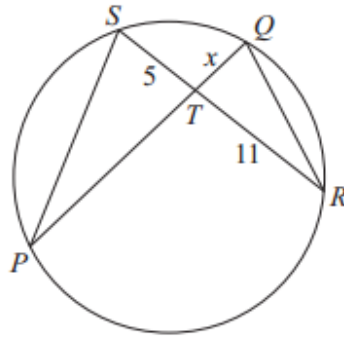
$ST = 5$ cm, $TR = 11$ cm and $TQ = x$ cm.

Given that $PQ = 18$ cm, show that x satisfies the equation

$$x^2 - 18x + 55 = 0.$$

[2]

(iii) Solve the equation $x^2 - 18x + 55 = 0$.
Give each solution correct to 1 decimal place.

Answer $x =$ .... or .... [3]

(iv) Find the difference between the lengths of PT and TQ.

Answer .... cm [1]

Q9

10 A fuel tanker delivers fuel in a cylindrical container of length 9.5 m and radius 0.8 m.

(a) After several deliveries, the fuel remaining in the container is shown in the diagram.
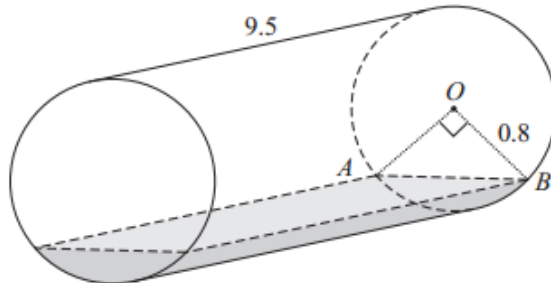
AB is horizontal, *O* is the centre of the circular cross-section and $A\hat{O}B = 90°$.

(i) Calculate the curved surface area of the container that is in contact with the fuel.

Answer .... m^2 [2]

(ii) Calculate the volume of fuel remaining in the container.

Answer .... m^3 [4]

(iii) Calculate this volume remaining as a percentage of the volume of the whole container.

Answer .... % [2]

(b) The fuel is pumped through a cylindrical pipe of radius 4.5 cm at a rate of 300 cm/s.

  (i) Calculate the volume pumped in 1 second.

  Answer .... cm^3 [1]

  (ii) Calculate the time taken, in minutes, to pump 25 000 litres of fuel.
  Give your answer correct to the nearest minute.

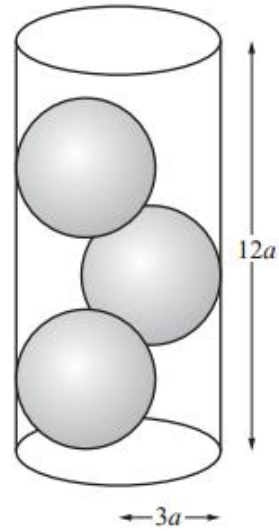 *Answer* .... minutes [3]

Q10

16 [Volume of a sphere = $\frac{4}{3}\pi r^3$]

Three spheres, each of radius $2a$ cm
are placed inside a cylinder of
radius $3a$ cm and height $12a$ cm.

Water is poured into the cylinder to fill it
completely.

The volume of water is $k\pi a^3$ cm^3.

Find the value of k.

12a

3a

Answer $k =$ .... [3]

Q11

19 P is $(-1, 3)$ and Q is $(5, -1)$.

(a) Find the coordinates of the midpoint of PQ.

Answer (............... , ...) [1]

(b) Find the gradient of the line PQ.

Answer .... [1]

(c) Given that the length of $PQ = 2\sqrt{n}$ units, where n is an integer, find the value of n.

Answer $n =$ .... [2]

Q12

22
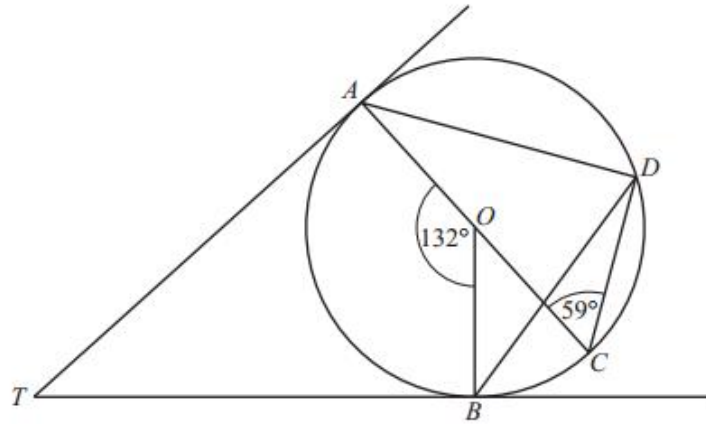
In the diagram, the points A, B, C and D lie on the circle centre O.
TA and TB are tangents touching the circle at A and B respectively.

$A\hat{O}B = 132°$, $A\hat{C}D = 59°$ and AOC is a straight line.

(a) Find $A\hat{T}B$.

Answer $A\hat{T}B =$ .... [1]

(b) Find $B\hat{D}A$.

Answer $B\hat{D}A =$ .... [1]

(c) Find $B\hat{D}C$.

Answer $B\hat{D}C =$ .... [1]

(d) Find $O\hat{B}D$.

Answer $O\hat{B}D =$ .... [1]

Q13

23
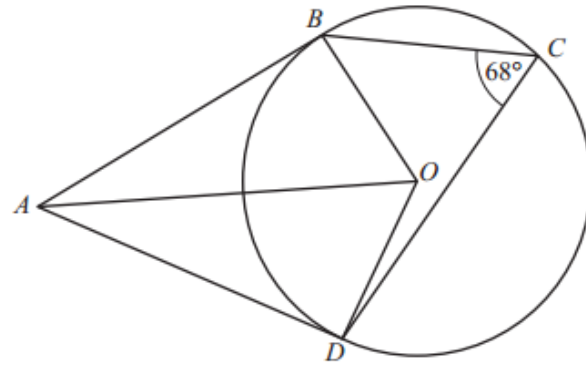
B, C and D are points on the circle, centre O.
BA and DA are tangents to the circle at B and D.

(a) Show that triangles ABO and ADO are congruent.

....

....

....

....

....

.... [3]
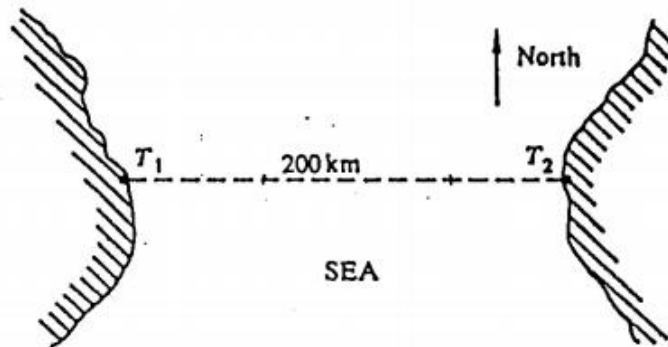
Q14

6

T_1 and T_2 are transmitters 200 km apart. T_2 is due East of T_1.

The signals from T_1 can reach a distance of 150 km and those from T_2 can reach a distance of 120 km.

(a) Using a scale of 1 cm to represent 20 km, make an accurate drawing to represent the transmitters and the area where signals from both transmitters can reach. [3]

(b) A ship is sailing on a bearing of 330° and passes through the point exactly half-way between the two transmitters.

On the same drawing, show accurately the path of the ship. [3]

(c) Use your drawing to find the distance the ship sails whilst receiving signals from both transmitters. [2]

(d) Given that the speed of the ship is 25 km/h, calculate the length of time during which the ship can receive signals from both transmitters. [2]


Q15

10 Green Line buses run every 10 minutes.
Red Line buses run every 20 minutes.
Purple Line buses run every 35 minutes.
One bus from each Line leaves the city centre at 09 00.
After how many minutes will buses from all three Lines next leave the city centre at the same time?

Answer .... [2]

Q16

11   Similar buckets are available in two sizes.
      The large bucket has height 30 cm and base diameter 16 cm.
      The small bucket has base diameter 8 cm.
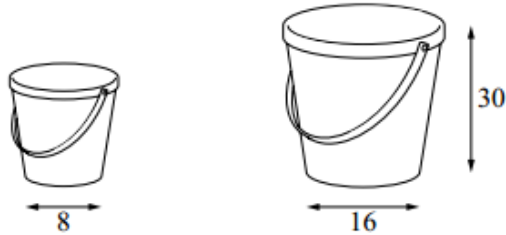
(a) Find the height of the small bucket.
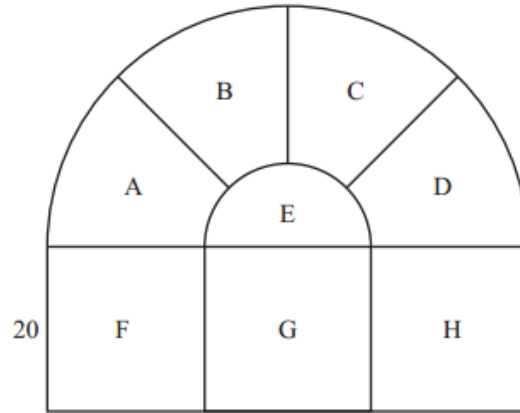
Answer (a) .... cm [1]

(b) Given that the small bucket has volume 850 cm^3, find the volume of the large
     bucket.

Answer (b) .... cm^3 [2]

Q17

22 The diagram shows a window made up of a large semicircle and a rectangle.
The large semicircle has 4 identical sections, A, B, C, D, and a small semicircle, E.
The rectangle has three identical square sections, F, G and H.
The side of each square is 20 cm.

Find an expression, in the form $a + b\pi$, for

(a) the area of the whole window,

Answer ....cm^2 [2]

(b) the perimeter of section B.

Answer ....cm [3]

Q18

16 (a) The profits of a company were $5 million in 2009 and $8 million in 2010.

Find the percentage increase in profits from 2009 to 2010.

Answer ....% [1]

(b) Another company had an income of $20 million in 2008.
In 2009 this income decreased by 10%.
In 2010 the income increased by 15% from the 2009 income.

Find the income in 2010.

Answer $ .... million [2]

Q19

2 (a) Sunil needs to hire a digger from Monday to Thursday one week and on Monday and Tuesday the following week.
The Hire company charges $48 each time the digger is hired plus $13 per day.
He has two options.

Option 1: Hire the digger for four days, return it and then hire it again for two days.

Option 2: Hire it continuously from the first Monday to the second Tuesday.

Which is the cheaper option and by how much?

Answer Option .... is cheaper by $ .... [2]

(b) Tina invests some money in an account that earns simple interest at 3% per year.
At the end of one year the investment is worth $2781.

How much money did she invest?

Answer $.. [2]